A Joint Solution for Scheduling and Precoding in Multiuser MISO Downlink Channels

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Abstract

The long-term average performance of the MISO downlink channel, with a large number of users compared to transmit antennas of the base station, depends on the interference management which necessitates the joint design of scheduling and precoding. Unlike the previous works which do not offer a truly joint design, this paper focuses on formulating a problem amenable for the joint update of scheduling and precoding. Novel optimization formulations are investigated to reveal the hidden difference of convex/ concave structure for three classical criteria (weighted sum rate, max-min signal-to-interference plus noise ratio, and power minimization) and associated constraints are considered. Thereafter, we propose a convex-concave procedure framework based iterative algorithm where scheduling and precoding variables are updated jointly in each iteration. Finally, we show the superiority in performance of joint solution over the state-of-the-art designs through Monte-Carlo simulations.

Index Terms

User scheduling, Precoding, multiuser, difference-of-convex, convex-concave procedure

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I. Introduction

With the adoption of full frequency reuse in the next-generation cellular networks, interference among the simultaneously served users becomes a limiting factor thwarting the achievement of near-optimal capacity [2]–[5]. Linear precoding has been largely used to achieve satisfactory interference mitigation at low complexity [6], [7]. Moreover, in a network with a large number of users compared to the number of BS transmit antennas, user scheduling for simultaneous transmission is pivotal for interference management [8], [9]. Optimizing performance in such a network involves the design of precoding variables and user scheduling. Further, different perspectives to network performance motivate the need to investigate multiple figures of merit; these include network throughput, user Quality of Service (QoS) power consumed among others. In this context, we address the joint design of scheduling and precoding problem for multiuser MISO downlink channels in single-cell scenario for the following network optimization design criteria: 1) Maximize the weighted sum rate subject to user's minimum signal-to-interference plus noise ratio (MSINR), scheduling and power constraints referred to in the sequel simply as WSR. 2) Maximize the MSINR of the scheduled users subject to scheduling and total power constraints henceforth referred to as MMSINR. 3) Minimize the power utilized subject to scheduling and MSINR constraints henceforth referred to as PMIN.

The aforementioned criteria are designed to improve the complementary aspects of the networks. In all practical wireless systems, a certain minimum received SINR is required for the successful transmission of information. In light of this, to enhance the practical relevance, SINR constraints are introduced in these design criteria. The WSR problem improves the overall throughput of a network while satisfying the scheduling constraint and QoS requirement on the scheduled users. On the contrary, the MMSINR problem improves the performance of the poorest user (in terms of SINR) among those scheduled. Unlike WSR and MMSINR, PMIN optimizes the consumed power while meeting the scheduling and SINR constraints. An elaborate discussion on each design is provided in the subsequent sections.

The joint design of scheduling and precoding, which we simply refer to as joint design, is well studied during the last decade (see [10] and references therein). Most of the existing literature on the joint design can be classified as:

- *Non-iterative decoupled approach*: In this approach, scheduling and precoding are treated as two decoupled problems where usually the users are scheduled according to some criteria followed by precoding [8], [9], [11]–[13].

- *Iterative decoupled approach*: In this approach, scheduling and precoding are still treated as two separate problems. However, scheduling and precoding parameters are refined in each iterate to improve the objective based on the feedback from the previous iterate [14]–[17].
- *Joint formulation with alternate update*: In this approach, the joint design problem is formulated as a function of both scheduling and precoding [18]–[20]. However, these formulations are not amenable for the joint update as the scheduling variables are coupled to precoding variables which inhibits their joint update. Hence, during the solution stage either scheduling constraints are ignored [18] or the scheduling and precoding variables are updated alternatively. [19].

The joint design is a coupled problem where the efficiency of the precoder design depends on the interference among the users which, in turn, is a function of the scheduled users [10]. Hence, the joint update of scheduling and precoding has the potential to achieve better performance over the aforementioned approaches [8], [9], [18], [11]–[17]. The authors in [21] shown the user scheduling and power allocation problem to be NP-hard. The joint design problems that are considered in this work encapsulate the problem considered in [21] as a special case. Hence, the considered joint design problems are NP-hard. It is also non-convex due to the constraints on the SINR or rate of scheduled users [14]. Hence, the optimal solution entails an exhaustive search over Boolean space (user scheduling) and further involves the solution of a non-convex precoding problem. The exponential complexity of an exhaustive search for practical system dimensions motivates a shift towards low-complexity achievable solutions. In this context, we quickly review the various relevant works to place ours in perspective.

The joint design problem to maximize the weighted sum rate subject to total power constraint, which is referred to as the classical WSR problem, is considered for single cell networks in [8], [9], [11]. The channel orthogonality based scheduling followed by zero-forcing precoding (SUS-ZF) proposed in [9] is proven to be asymptotically optimal for sum rate maximization. However, it is easy to see that SUS-ZF is not optimal for WSR with non-uniform weights and QoS constraints. Similarly, the classical WSR is addressed for multicell networks in [14], [16], [17] and hierarchical networks in [18]. The joint design problem is also considered for MMSINR in [13] and PMIN in [15]. However, scheduling and precoding are not jointly updated in the aforementioned works.

The coupled nature of binary variables with precoding vector appears in many other formulations [22], [23] etc. For example, towards maximizing the weighted sum-rate in a hierarchical network, binary variables associated with users get multiplied to signal power and interference power of SINR [18]. Similarly, in [20], a binary variable is multiplied to the rate of the users in the weighted sum-rate maximization problem. Please note that system models and objectives discussed in [18], [20], [23] are different from each other, and the emphasis is only on the occurrence of the joint design (coupled discrete and continuous) nature that prevails in different designs. The multiplicative nature in previous formulations precludes the joint update of scheduling and precoding. To the best of our knowledge, no prior work exists that update the scheduling and precoding jointly for the aforementioned WSR, MMSINR and PMIN problems. Therefore, we focus on formulating the joint design problem for WSR, MMSINR, and PMIN that facilitates the joint scheduling and precoding solutions.

Revisiting the WSR and MMSINR design problems for fixed scheduled users, it is well-known that the problems are non-convex with difficulty to obtain a global solution. However, efficient suboptimal solutions have been proposed for WSR in [24] and MMSINR in [25], [26] by formulating these as difference-of-convex (DC) programming problems with the help of auxiliary variables and semidefinite programming (SDP) transformations and relaxations. However, the semidefinite relaxations for WSR and MMSINR often lead to non-unity rank solutions from which the approximate rank-1 solutions are extracted [24]–[26]. The rank-1 approximation results in a loss of performance. Moreover, the transformed problems have higher complexity than the original problems due to auxiliary variables and SDP transformations. In this work, we pose the WSR and MMSINR joint design as DC programming problems without SDP transformation and employing a minimal number of auxiliary variables.

The aforementioned discussion reflects on the novelties of the paper-based both on problem formulation and its solution. The contributions of the paper include:

- The scheduling is handled through the power of the precoding vector of the corresponding user, where non-zero power indicates the user being scheduled (and not scheduled otherwise). Unlike the previous works [18], [20], [23], a binary variable is used for upper bounding the power of the precoding vector. This renders the formulation amenable to the joint design of scheduling and precoding.

- With the help of the aforementioned scheduling, the joint design problem for WSR, MM-SINR, and PMIN design criteria are formulated as mixed-integer non-linear programming (MINLP) in a way that would facilitate the joint updates of scheduling and precoding. Here, the nonconvexity of the problem stems from rate and SINRs in the objective and constraints.
- The binary nature of the problem due to scheduling constraints is addressed by relaxing the binary variables into real values. This is followed by penalizing the objective with a novel entropy-based penalty function to promote a binary solution for the scheduling variables. This step transforms the optimization into a continuous non-convex problem.
- Unlike the classical DC formulation using SDP transformation [24]–[26], a novel useful reformulation of the objective and/or SINR constraints are proposed to manipulate the joint design as DC programming without SDP transformation.
- Further, a convex-concave procedure (CCP) based low-complexity iterative algorithm is proposed for WSR, MMSINR and PMIN DC problems. A procedure is proposed to find a feasible initial point, which is sufficient for these algorithms to converge to a stationary point [27].
- Subsequently, the per iteration complexity of the CCP based algorithms, is discussed. Finally, the efficiency of the proposed DC reformulations is compared to the decoupled solutions using the Monte-Carlo simulations.

The rest of the paper is organized as follows. Section II presents the system model and problem formulation of WSR, MMSINR, and PMIN problem. The reformulations and algorithm are proposed for WSR in Section III, MMSINR in Section IV and PMIN in Section V respectively. Section VI presents simulation results, followed by conclusions in Section VII.

Notation: Lower or upper case letters represent scalars, lower case boldface letters represent vectors, and upper case boldface letters represent matrices. $\|\cdot\|$ represents the Euclidean norm, $|\cdot|$ represents the cardinality of a set or the magnitude of a scalar, $(\cdot)^H$ represents Hermitian transpose, $(\cdot)^T$ represents transpose, $\binom{a}{b}$ represents a choose b, $\mathrm{tr}\{\}$ represents trace and $\mathbb{R}\{\}$ represents real operation, $\mathbb{E}\{\}$ represents expectation operator and s.t. is referred to as subject to and ∇ represents the gradient.

II. System Model

Consider the downlink transmission of a single cell MISO system with N users in a cell and a BS with $M(\leq N)$ antennas. Let $\mathbf{h}_i \in \mathbb{C}^{M\times 1}$, $\mathbf{w}_i \in \mathbb{C}^{M\times 1}$ and x_i denote the downlink channel, precoding vector and data of user i respectively. The BS is assumed to transmit independent data to utmost M among N users and $\mathbb{E}\{|x_i|^2\} = 1, \forall i$. Further, let n_i be the noise at user i; the noise realizations at all users are assumed to be independent and characterized as additive white complex Gaussian with zero mean and variance σ^2. Let y_i be the noisy received signal of the user i and $\mathbf{y} \triangleq [y_1, \dots y_N]^T$. The generative model of the received signal vector $\mathbf{y}$ of all users is given by,

$$\mathbf{y} = \mathbf{H}\mathbf{W}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{H} \triangleq [\mathbf{h}_1, \dots, \mathbf{h}_N]^H$, $\mathbf{W} \triangleq [\mathbf{w}_1, \dots, \mathbf{w}_N]$, $\mathbf{x} \triangleq [x_1, \dots x_N]^T$, $\mathbf{n} \triangleq [n_1, \dots n_N]^T$.

Hence, this leads to the scheduling of the utmost M users for WSR and exactly M users for MMSINR and PMIN. Further, it is assumed that perfect channel state information of all the users is available at BS and that the user channels are constant during the transmission.

Towards defining the WSR problem mathematically, let $\mathcal{T} = \{1, \dots N\}$ be the set containing indices of all users and $\bar{\mathcal{K}}$ be a subset of $\mathcal{T}$ with cardinality less than or equal to M. Further, let $\mathcal{K}$ be the collection of all the possible subsets of type $\bar{\mathcal{K}}$; clearly, the cardinality of $\mathcal{K}$ is $C \triangleq \sum_{i=0}^{M} \binom{N}{i}$. With the notations defined, the joint design problem to maximize the WSR subject to constraints on the minimum SINR of the scheduled users and total consumed power is defined as,

$$\mathcal{P}_{\text{WSR}}: \quad \underbrace{\max_{\forall \bar{\mathcal{K}} \in \mathcal{K}} \quad \underbrace{\begin{aligned} &\max_{\mathbf{W}_{\bar{\mathcal{K}}}} \sum_{\forall i \in \bar{\mathcal{K}}} \alpha_i R_i \\ &\text{s.t } R_i \geq \tilde{\epsilon}_i, \forall i \in \bar{\mathcal{K}}, \\ &\sum_{\forall i \in \bar{\mathcal{K}}} \|\mathbf{w}_i\|_2^2 \leq P_T, \end{aligned}}_{\text{precoding problem for selected users}}}_{\text{Joint schedueling and Precoding problem}} \tag{2}$$

where $\gamma_i \triangleq \dfrac{|\mathbf{h}_i^H \mathbf{w}_i|^2}{\sigma^2 + \sum_{j \neq i \in \bar{\mathcal{K}}} |\mathbf{h}_i^H \mathbf{w}_j|^2}$, $R_i \triangleq \log_2(1+\gamma_i)$ and $\epsilon_i \geq 0$ are the SINR, rate and minimum rate requirement of the user i respectively and $\bar{\mathcal{K}}$ is set of scheduled users. Further $\alpha_i \in \mathcal{R}^+$ denotes the weight for ith user offering design flexibility, P_T is the total available power, and $\mathbf{W}_{\bar{\mathcal{K}}} = \{\mathbf{w}_i\}_{i=1}^{|\bar{\mathcal{K}}|}$ is the precoding matrix containing the precoding vectors of users belonging to set $\bar{\mathcal{K}}$.

Unlike the WSR design, scheduling of exactly $K(\leq M)$ users is considered in MMSINR formulation. This is because constraining the scheduling to utmost K users always leads to the trivial solution of scheduling only one user and an elaborate discussion is provided at the beginning of Section IV. Let $\bar{\mathcal{S}}$ be a subset of $\mathcal{T}$ with cardinality equal to K. Let $\mathcal{S}$ be the collection of all the possible subsets of type $\bar{\mathcal{S}}$; clearly, the cardinality of $\mathcal{S}$ is $\binom{N}{K}$. Letting $\tilde{\epsilon}_i$ to the minimum SINR requirement of user i, $\forall i$, the design problem for MMSINR can then be defined as,

$$\mathcal{P}_{\text{MMSINR}}: \underbrace{\max_{\bar{\mathcal{S}} \subseteq \mathcal{S}} \quad \max_{\mathbf{W}_{\bar{\mathcal{S}}}} \quad \min_{i \subseteq \bar{\mathcal{S}}} \; \{\beta_i \gamma_i\} \quad \text{s.t} \sum_{i \in \bar{\mathcal{S}}} \|\mathbf{w}_i\|_2^2 \leq P_T, \quad \underbrace{\gamma_i \geq \tilde{\epsilon}_i, i \in \bar{\mathcal{S}},}_{\text{precoding problem for selected users}}}_{\text{Joint schedueling and Precoding problem}} \tag{3}$$

where $\beta_i \in \mathcal{R}^+$, is weight and $\mathbf{W}_{\bar{\mathcal{S}}} = \{\mathbf{w}_i\}_{i=1}^{|\bar{\mathcal{S}}|}$ is the matrix containing the precoding vectors of users in the set $\bar{\mathcal{S}}$. Notice that to accommodate the fairness in the designs, weights or priority factors are introduced through α and β in WSR and MMSINR problems respectively. Various fairness metrics are proposed in the literature, e.g. fairness in terms of rates and allocated power are considered at the physical layer. We refer to [28] and references therein for details on fairness.

Finally, towards defining the PMIN problem, scheduling exactly $K(\leq M)$ users is considered for the same reason mentioned in MMSINR. With notations defined for MMSINR criteria, the PMIN problem is defined as:

$$\mathcal{P}_{\text{PMIN}}: \underbrace{\min_{\bar{\mathcal{S}} \subseteq \mathcal{S}} \underbrace{\min_{\mathbf{W}_{\bar{\mathcal{S}}}} \sum_{i \in \bar{\mathcal{S}}} \|\mathbf{w}_i\|_2^2 \;\text{ s.t. } \gamma_i \geq \tilde{\epsilon}_i, \; i \subseteq \bar{\mathcal{S}}.}_{\text{PMIN problem for selected users}}}_{\text{Joint user scheduling and PMIN problem}} \tag{4}$$

The inner optimization in (2), (3), and (4) solves the precoding problem for the scheduled users. The outer optimization, on the other hand, ensures scheduling users with a maximum objective value among all scheduling possibilities. Notice that the inner and outer optimization are coupled

- the design of precoder depends on the selected set of users, while the user scheduling depends on the objectives in (2), (3) and (4) which, in turn, are functions of the precoder [29].

Towards proposing low-complexity algorithms, we begin by addressing the user scheduling through the precoding vectors. Accordingly, user i is not scheduled if the norm of the corresponding precoding vector is zero i.e,

$$\|\mathbf{w}_i\|_2 = \begin{cases} = 0; \text{user not selected,} \\ \neq 0; \text{user selected.} \end{cases} \tag{5}$$

The zero norm of the precoding vector $\mathbf{w}_i$ ensures that all elements of $\mathbf{w}_i$ are zero. Hence, the user i is not scheduled. Similarly, the non-zero norm of the precoder vector $\mathbf{w}_i$ indicates user i being scheduled with an assigned power of $\|\mathbf{w}_i\|_2^2$. In the sequel, we focus on the design of low-complexity solutions to the joint design using (5) to achieve better performance than the decoupled designs.

III. Weighted Sum Rate maximization

In (2), the weighted sum rate objective is considered to improve the overall weighted throughput of the network. Thus, WSR problem schedules only the users who contribute to maximizing the objective. Given sufficient resources, the WSR design schedules close to M users as the objective increases linearly with the number of scheduled users; on the other hand, scheduling of few users with high SINRs only contributes logarithmically to the objective. Hence, the constraint of scheduling utmost of M users is considered as opposed to the harder constraint of scheduling to exactly M users. Besides, the design is flexible to favor users by increasing the corresponding weights i.e., α_i to relatively larger values over the users. The minimum rate constraints preclude scheduling of the users whose rates are not in the range of interest. Since the scheduling of zero users in also included in the feasible set, the problem (2) is always feasible. In the sequel, the WSR problem (i.e., (2)) is transformed as a DC programming problem through a sequence of novel reformulations and low-complexity sub-optimal algorithms within the framework of CCP.

A. Joint Design Problem Formulation: WSR

Letting $\bar{\mathcal{K}}$ to be the set of scheduled users, a tractable formulation of (2) using (5) is,

$$\mathcal{P}_1^{\text{WSR}} : \max_{\mathbf{W}, \forall \bar{\mathcal{K}} \in \mathcal{K}} \sum_{i=1}^{N} \alpha_i R_i \tag{6}$$

$$\text{s.t. } C_1: \left\| \left[\|\mathbf{w}_1\|_2, \ldots, \|\mathbf{w}_N\|_2 \right] \right\|_0 \leq M,$$
$$C_2: \sum_{i=1}^{N} \|\mathbf{w}_i\|_2^2 \leq P_T,$$
$$C_3: R_i \geq \epsilon_i,\ i \in \bar{\mathcal{K}}.$$

Remarks:

- It is clear from (5) and the definition of ℓ_0 norm, that the constraint C_1 imposes restrictions on the total number of selected users to utmost M. We refer to this constraint as the user scheduling constraint throughout this section.
- The constraint C_2 precludes the design from using a transmission power greater than P_T.
- The constraint C_3 imposes a minimum rate required for the scheduled users.

A Novel MINLP formulation: The problem $\mathcal{P}_1^{\text{WSR}}$ is combinatorial due to the constraint C_1 and C_3, and non-convex due to the objective and constraints C_1 and C_3. Towards addressing the combinatorial nature, we let η_i to be the binary scheduling variable associated with user i, $\boldsymbol{\eta} = [\eta_1, \ldots, \eta_N]^T$ and $\tilde{\epsilon}_i \triangleq 2^{\epsilon_i} - 1, \forall i$. Similarly, Let ζ_i to be the slack variable associated with user i and $\boldsymbol{\zeta} = [\zeta_1, \ldots, \zeta_N]^T$. With the defined notations, a tractable formulation of C_1 and C_3 of $\mathcal{P}_1^{\text{WSR}}$ then takes the form,

$$\mathcal{P}_2^{\text{WSR}}: \max_{\mathbf{W},\boldsymbol{\zeta},\boldsymbol{\eta}} f\left(\boldsymbol{\zeta}, \boldsymbol{\eta}\right) \triangleq \sum_{i=1}^{N} \alpha_i \log\left(\zeta_i\right) \tag{7}$$
$$\text{s.t. } C_1: \eta_i \in \{0, 1\},\ \forall i,$$
$$C_2: \|\mathbf{w}_i\|_2^2 \ \leq\ P_T \eta_i,\ \forall i,$$
$$C_3: \sum_{i=1}^{N} \eta_i \leq M,$$
$$C_4: \sum_{i=1}^{N} \|\mathbf{w}_i\|_2^2 \leq P_T,$$
$$C_5: 1 + \gamma_i \geq \zeta_i, \forall i,$$
$$C_6: \zeta_i \geq 1 + \eta_i \tilde{\epsilon}_i,\ \forall i,$$

Remarks:

- The binary nature of η_i (i.e., C_1) together with C_2 determines the scheduling of users. In other words, $\eta_i = 0$ leads to a precoding vector containing all zero entries. Similarly $\eta_i = 1$

leads to $\|\mathbf{w}_i\|_2^2 \leq P_T$ which is a trivial upper bound compared to C_4. Hence the constraint C_2 along with C_1 contributes only to the scheduling aspects of the problem.

- From the objective and constraint C_5, the variable ζ_i provides a lower bound for $1+\gamma_i$.
- The constraint C_6 ensures minimum SINR or rate constraint of the scheduled users. If user i is scheduled i.e., $\eta_i = 1$, from C_6, $\zeta_i \geq \tilde{\epsilon}_i$. Similarly, for an unscheduled user i, C_6 becomes $\zeta_i \geq 0$. In fact for $\eta_i = 0$, constraint is met with equality i.e., $\zeta_i = 0$ due to C_2.
- It is easy to see that, at the optimal solution, the constraints C_5 and C_6 are met with equality.

Novelty of $\mathcal{P}_2^{\text{WSR}}$: Novelty of $\mathcal{P}_2^{\text{WSR}}$ lies in the formulation of scheduling constraint, C_2. This reformulation is vital to the facilitation of the joint update of $\boldsymbol{\eta}$ and $\mathbf{W}$ as discussed in the sequel. Notice that this formulation differs from those in the literature ([18], [20], [23], [30], [31], etc) where the scheduling constraint is handled by a binary slack variable which multiplies either the precoding vector or the rate of the user, to control the user scheduling. This multiplication not only makes the constraints non-convex but also makes it difficult to obtain the joint update of Boolean and continuous variables due to the coupling of variables. Moreover, the constraints C_5 and C_6 help to reformulate the objective as a concave function and connects the minimum rate constraints to the objective. This reformulation is crucial as it facilitates the reformulation of $\mathcal{P}_2^{\text{WSR}}$ as DC programming problem without resorting to SDP transformations [24], [32]–[34].

B. A Novel DC reformulation: WSR

A novel rearrangement of SINR constraint C_5 in $\mathcal{P}_2^{\text{WSR}}$ that transforms $\mathcal{P}_2^{\text{WSR}}$ as a DC programming problem without SDP transformation is,

$$\mathcal{P}_3^{\text{WSR}} : \max_{\mathbf{W},\boldsymbol{\zeta},\boldsymbol{\eta}} f(\boldsymbol{\zeta},\boldsymbol{\eta}) \triangleq \sum_{i=1}^{N} \alpha_i \log(\zeta_i) \tag{8}$$

$$\text{s.t. } C_1, C_2, C_3, C_4 \text{ and } C_6 \text{ in (7)}$$

$$C_5 : \mathcal{I}_i(\mathbf{W}) - \mathcal{G}_i(\mathbf{W},\zeta_i) \leq 0, \forall i,$$

where $\mathcal{I}_i(\mathbf{W}) = \sigma^2 + \sum_{j\neq i}|\mathbf{h}_i^H\mathbf{w}_j|^2$ and $\mathcal{G}_i(\mathbf{W},\zeta_i) = \dfrac{\sigma^2 + \sum_{j=1}^{N}|\mathbf{h}_i^H\mathbf{w}_j|^2}{\zeta_i}$. Notice that $\mathcal{I}_i(\mathbf{W})$ is convex in $\mathbf{W}$, and for $\zeta_i > 0$, $\mathcal{G}_i(\mathbf{W},\zeta_i)$ is also jointly convex in $\mathbf{W}$ and ζ_i. Hence, (8) is a DC programming problem with combinatorial constraint C_1. This is the first attempt at reformulating the WSR towards a tractable form without resorting to SDP methods or use of additional slack variables thereby rendering a low-complexity solution to the problem.

Beyond SDP based DC formulation: Notice that for fixed $\boldsymbol{\eta}$, the problem $\mathcal{P}_3^{\text{WSR}}$ becomes a classical WSR maximization problem subject to SINR and total power constraints [24], [32]–[34]. The problem $\mathcal{P}_3^{\text{WSR}}$ is non-convex due to the constraint C_5. Although, for fixed $\boldsymbol{\zeta}$ (i.e. fixed $\boldsymbol{\eta}$), the constraint C_5 in $\mathcal{P}_3^{\text{WSR}}$ is formulated as a second-order cone programming (SOCP) constraint [6], a similar SOCP transformation of C_5 is not known when $\boldsymbol{\zeta}$ is variable. On the other hand, many previous works have exploited the DC structure in WSR maximization problem without SINR constraint in [32]–[34] and with SINR constraint in [24] by transforming it into an SDP problem. However, the SDP transformations in [24], [32]–[34], essentially increase the number of variables, thereby increasing the complexity. Moreover, SDP transformations also introduce the non-convex rank-1 constraint on the solutions which is difficult to handle in general; this has led to semidefinite relaxations [7] followed by extraction of approximate feasible rank-1 solutions.

The problem $\mathcal{P}_3^{\text{WSR}}$ is still an MINLP with a DC structure in the non-convexity. This structure can be leveraged with the optimization tools like CCP. Now, to circumvent the combinatorial nature of $\mathcal{P}_3^{\text{WSR}}$, η_i is relaxed to a box constraint between 0 and 1, and penalized with $\mathbb{P}\left(\eta_i\right)$ so that the relaxed problem favours 0 or 1. The penalized reformulation of $\mathcal{P}_3^{\text{WSR}}$ with penalty parameter $\lambda_1 \in \mathcal{R}^+$ is,

$$\mathcal{P}_4^{\text{WSR}}: \max_{\mathbf{W},\boldsymbol{\eta},\boldsymbol{\zeta}} \sum_{i=1}^{N} \big(\alpha_i \log\left(\zeta_i\right) + \lambda_1 \mathbb{P}\left(\eta_i\right)\big) \tag{9}$$

$$\text{s.t. } C_1: 0 \leq \eta_i \leq 1, \ \forall i,$$

$$C_2, C_3, C_4, C_5 \text{ and } C_6 \text{ in (8).}$$

We propose a new penalty function $\mathbb{P}(\eta_i) \triangleq \eta_i \log \eta_i + (1-\eta_i) \log\left(1-\eta_i\right)$ which is a convex function in $\eta_i \geq 0$. $\mathbb{P}(\eta_i)$ incurs no penalty at $\eta_i = 0$ or 1 and the penalty increases logarithmically as η_i drifts away from $\eta_i = 0$ or 1 with the highest penalty at $\eta_i = 0.5$. Hence, by choosing λ_1 appropriately, binary nature of $\boldsymbol{\eta}$ is ensured.

Now, notice that the objective in $\mathcal{P}_4^{\text{WSR}}$ a difference of concave functions i.e. $f\left(\boldsymbol{\zeta}, \boldsymbol{\eta}\right) = \sum_{i=1}^{N} \big(\alpha_i \log\left(\zeta_i\right)\big) - \Big(-\sum_{i=1}^{N} \lambda_1 \mathbb{P}\left(\eta_i\right)\Big)$ and constraints are convex and DC. Hence, the problem $\mathcal{P}_4^{\text{WSR}}$ is a DC programming problem. In the sequel, a CCP based algorithm is proposed [35].

C. JSP-WSR: A Joint Design Algorithm

In this section, we propose a CCP based iterative algorithm to the DC problem in (9) which we refer to as *JSP-WSR*. CCP is a powerful tool to find a stationary point of DC programming problems. Within this framework, an iterative procedure is performed, wherein the two steps of Convexification and Optimization are executed in each iteration. In the convexification step, a concave optimization problem is obtained from $\mathcal{P}_4^{\text{WSR}}$ by linearizing the objective and constraints. Hence, by definition, the modified objective and constraints lower bound the actual objective and constraints of $\mathcal{P}_4^{\text{WSR}}$ where the lower bound is tight at the previous iteration [35]. The optimization step then solves the convex sub-problem globally. Thus, the proposed JSP-WSR algorithm iteratively executes the following two steps until convergence:

- Convexification: Let $(\mathbf{W}, \boldsymbol{\eta}, \boldsymbol{\zeta})^{k-1}$ be the estimates of $\mathbf{W}, \boldsymbol{\eta}, \boldsymbol{\zeta}$ in iteration $k-1$ and $\mathcal{G}_i(\mathbf{W}, \zeta_i)$. In iteration k, the convex part of the objective in $\mathcal{P}_4^{\text{WSR}}$ i.e., $\sum_{i=1}^{N} \lambda_1 \mathbb{P}\left(\eta_i\right)$, and the concave part of constraint C_5 in $\mathcal{P}_4^{\text{WSR}}$ for user i are replaced by their first order Taylor approximations around the estimate of $(\mathbf{W}, \boldsymbol{\eta}, \boldsymbol{\zeta})^{k-1}$

$$\tilde{\mathbb{P}}\left(\eta_i\right) \triangleq \lambda_1 \left(\mathbb{P}\left(\eta_i^{k-1}\right) + \left(\eta_i - \eta_i^{k-1}\right) \nabla \mathbb{P}\left(\eta_i^{k-1}\right)\right),$$

$$\tilde{\mathcal{G}}_i(\mathbf{W}, \zeta_i)^{k-1} \triangleq -\mathcal{G}_i(\mathbf{W}, \zeta_i) - \mathbb{R}\left\{ \nabla^H \mathcal{G}_i(\mathbf{W}, \zeta_i)^{k-1} \begin{bmatrix} \mathbf{w}_1 - \mathbf{w}_1^{k-1} \\ \vdots \\ \mathbf{w}_N - \mathbf{w}_N^{k-1} \\ \zeta i - \zeta_i^{k-1} \end{bmatrix} \right\}, \tag{10}$$

where

$$\nabla \mathcal{G}_i(\mathbf{W}, \zeta_i)^{k-1} = \begin{bmatrix} \dfrac{2\mathbf{h}_i \mathbf{h}_i^H \mathbf{w}_1^{k-1}}{\zeta_i^{k-1}} \\ \vdots \\ \dfrac{2\mathbf{h}_i \mathbf{h}_i^H \mathbf{w}_N^{k-1}}{\zeta_i^{k-1}} \\ -\dfrac{\sigma^2 + \sum_{j=1}^{N} |\mathbf{h}_i^H \mathbf{w}_j^{k-1}|^2}{\zeta_i^{k-1^2}} \end{bmatrix}. \tag{11}$$

- Optimization: The next update $(\mathbf{W}, \boldsymbol{\eta}, \boldsymbol{\zeta})^{k+1}$ is obtained by solving the following convex problem (which is obtained by replacing convex part of the objective and constraints in

$\mathcal{P}_4^{\text{WSR}}$ with (10) and ignoring the constant terms in the objective) :

$$\mathcal{P}_5^{\text{WSR}} : \max_{\mathbf{W},\boldsymbol{\zeta},\boldsymbol{\eta}} \sum_{i=1}^{N} \left(\alpha_i \log\left(\zeta_i\right) + \lambda_1 \eta_i \nabla \mathbb{P}\left(\eta_i^{k-1}\right) \right) \tag{12}$$

$$\text{s.t } C_1, C_2, C_3, C_4 \text{ and } C_6 \text{ in (9)}$$

$$C_5 : \mathcal{I}_i\left(\mathbf{W}\right) - \tilde{\mathcal{G}}_i(\mathbf{W}, \zeta_i) \leq 0, \forall i.$$

Remarks:

- Note that the proposed JSP-WSR algorithm is based on CCP framework hence a feasible initial point (FIP) is sufficient for the CCP procedure to converge to a stationary point (kindly refer [27]).
- Given the binary nature of $\boldsymbol{\eta}$ in the obtained stationary point, the converged stationary point is a valid feasible solution to the original problem $\mathcal{P}_1^{\text{WSR}}$. As mentioned previously, with appropriate λ_1 a stationary point with binary $\boldsymbol{\eta}$ can be obtained easily from the above iterative procedure.

In many cases, obtaining a FIP is difficult. However, in the next section, we propose a method which promises to obtain at least one FIP.

D. Feasible Initial Point: WSR

CCP is an iterative algorithm and an initial feasible point guarantees the solutions of all iterations remain feasible. A trivial initial FIP is obtained by the initializing $\{\mathbf{w}_i = \mathbf{0}\}_{i=1}^{N}, \boldsymbol{\eta} = \mathbf{0}$ and $\boldsymbol{\zeta} = \mathbf{1}$ where, $\mathbf{1}$ and $\mathbf{0}$ are the column vectors of length N with all ones and zeros respectively. Since the quality of the solution depends on the FIP, the harder task of finding a better FIP is considered through the following iterative procedure.

- Step 1: Initialize $\boldsymbol{\eta} = \hat{\boldsymbol{\eta}}$ that satisfies constraints C_1 and C_3 in $\mathcal{P}_4^{\text{WSR}}$, and $0 < \delta < 1$.
- Step 2: Solve the following optimization:

$$\mathcal{P}_{\text{FESWSR}} : \{\hat{\mathbf{W}}\} : \text{ find } \mathbf{W} \tag{13}$$

$$\text{s.t. } \tilde{C}_1 : \|\mathbf{w}_i\|_2^2 \;\leq\; \hat{\eta}_i P_T, \;\forall i,$$

$$\tilde{C}_2 : \left\| \left[\sigma \ldots \{\mathbf{h}_i^H \mathbf{w}_j\}_{j \neq i} \ldots \right] \right\|_2 \leq \frac{\mathbf{h}_i^H \mathbf{w}_i}{\sqrt{\hat{\eta}_i \tilde{\epsilon}_i}}, \;\forall i,$$

$$\tilde{C}_3 : \mathbb{R}\{\mathbf{h}_i^H \mathbf{w}_i\} \geq 0, \forall i,$$

$$\tilde{C}_4 : \Im\{\mathbf{h}_i^H \mathbf{w}_i\} == 0, \forall i,$$

$$\tilde{C}_5 : \|\mathbf{W}\|_2^2 \leq P_T.$$

- Step 3: If $\mathcal{P}_{\text{FESWSR}}$ is feasible go to step 4 else update $\boldsymbol{\eta} = \delta\hat{\boldsymbol{\eta}}$ and go to step 2.
- Step 4: Let $\hat{\mathbf{W}}$ be the solution of $\mathcal{P}_{\text{FESWSR}}$. Choose $\hat{\zeta}_i$ such that $1+\hat{\eta}_i\tilde{\epsilon}_i \leq \hat{\zeta}_i \leq 1+\hat{\gamma}_i$ where $\hat{\gamma}_i$ is the SINR of the user i calculated using $\hat{\mathbf{W}}$.

Remarks:

- Notice that the updates of $\hat{\boldsymbol{\eta}}$ are always feasible. Different $\hat{\boldsymbol{\eta}}$ in step 1 which satisfy the constraint C_1 and C_3 in $\mathcal{P}_4^{\text{WSR}}$ may lead to different FIPs. Similarly, different choices of $\delta \in (0,1)$ in step 1 may also lead to different FIPs.
- The optimization problem in Step 2 is only a function of $\mathbf{W}$ since $\boldsymbol{\eta}$ is fixed apriori and $\boldsymbol{\zeta}$ can be calculated easily from the solution of $\mathcal{P}_{\text{FESWSR}}$ as given in step 4.
- Following [6], the minimum SINR constraint, i.e. $\gamma_i \geq \hat{\eta}_i\tilde{\epsilon}_i$ is reformulated as a second-order cone (SOC) constraint as given in $\tilde{C}_2$ with the help of $\tilde{C}_3$ and $\tilde{C}_4$.
- If $\mathcal{P}_{\text{FESWSR}}$ in step 2 is in-feasible for $\boldsymbol{\eta}$ in step 1, update $\boldsymbol{\eta}$ as given in step 3 and repeat step 2. This is repeated until $\mathcal{P}_{\text{FESWSR}}$ in step 2 becomes feasible.
- If the initial iterates fail to result an non-zero based initial feasible point, the proposed method eventually lead to $\hat{\boldsymbol{\eta}} = \mathbf{0}$ and thus $\mathcal{P}_{\text{FESWSR}}$ in step 2 becomes feasible with $\hat{\mathbf{W}} = 0$. Hence, the proposed methods always results an FIP. By initializing $\hat{\boldsymbol{\eta}}$ close to $\mathbf{0}$, FIP can be obtained in fewer iterations.
- The FIP obtained by this procedure may not be feasible for the original WSR problem $\mathcal{P}_{\text{WSR}}$ in (2) unless $\mathcal{P}_{\text{FESWSR}}$ becomes feasible for $\{\hat{\eta}_i \in \{0,1\}\}_{i=1}^N$ satisfying $\sum_{i=1}^N \hat{\eta}_i \leq M$.
- Although the FIP obtained by this method is not feasible for $\mathcal{P}_{\text{WSR}}$, the final solution obtained by JSP-WSR with this FIP becomes a feasible for $\mathcal{P}_{\text{WSR}}$ since the solution satisfies the scheduling and SINR constraints of $\mathcal{P}_{\text{WSR}}$.

Letting $\mathcal{P}_5^{\text{WSR}}(k)$ be the objective value of the problem $\mathcal{P}_5^{\text{WSR}}$ at iteration k, the pseudo code of JSP-WSR for the joint design problem is given in algorithm 1.

E. Complexity: WSR

The computational complexity of JSP-WSR depends on the complexities of iterative procedures proposed in Section III-C and Section III-D. The proposed JSP-WSR in Section III-C is

Algorithm 1 JSP-WSR

Input: $\mathbf{H}, [\epsilon_1, \ldots, \epsilon_N], P_T, \Delta, \boldsymbol{\eta}^0, \mathbf{W}^0, \lambda_1 = 0, k = 1$

Output: $\mathbf{W}, \boldsymbol{\eta}$

while $|\mathcal{P}_5^{\text{WSR}}(k) - \mathcal{P}_5^{\text{WSR}}(k-1)| \geq \Delta$ **do**

  Convexification: Convexify the problem (10)

  Optimization: Update $(\mathbf{W}, \boldsymbol{\eta}, \boldsymbol{\zeta})^k$ by solving $\mathcal{P}_5^{\text{WSR}}$

  Update : $\mathcal{P}_5^{\text{WSR}}(k), \lambda_1, k$

end while

a CCP based iterative algorithm; hence, the complexity of the algorithm depends on complexity of the sub-problems $\mathcal{P}_5^{\text{WSR}}$. The convex problem $\mathcal{P}_5^{\text{WSR}}$ has $(NM + 2N)$ decision variables and $(2N + 1)$ convex constraints and $2N + 1$ linear constraints. Hence, the computational complexity of $\mathcal{P}_5^{\text{WSR}}$ is $\mathcal{O}\left((NM + 2N)^3 (4N + 2)\right)$ [36]. Similarly, the computational complexity of the proposed procedure in Section III-D to obtain a FIP depends on the per iteration complexity of $\mathcal{P}_{\text{FESWSR}}$. $\mathcal{P}_{\text{FESWSR}}$ is a convex problem with MN decision variables, $2N + 1$ convex constraints and $2N$ linear constraints. Hence, the computational complexity of $\mathcal{P}_{\text{FESWSR}}$ is $\mathcal{O}\left((MN)^3 (4N + 1)\right)$ [36].

IV. Max Min SINR

In this section, we focus on the development of a low-complexity algorithm for the MMSINR problem defined in (3). Dropping a user with low SINR clearly improves minimum SINR (MSINR). It also reduces the interference to the other users and the power of the dropped user can be used to further improve the MSINR of other users. Hence, the constraint of scheduling utmost K users leads to the global solution which has highest MSINR which is achieved by scheduling only one user. To avoid this, scheduling exactly K users is considered for MMSINR design. Besides the scheduling constraint, the minimum SINR requirements of the scheduled users are also considered. Without the minimum SINR requirement, the design becomes superficial as the solution might include zero SINR or SINR values which are not usable in practice.

Infeasibility of MMSINR: The infeasibility of the problem due to the minimum SINR requirement is explained in [6] for fixed set of users. Similarly, it may not be possible to find exactly K users while satisfying an arbitrarily chosen minimum SINR, power and system dimension constraints [6]; this renders the problem (3) infeasible. In this work, it is assumed that problem $\mathcal{P}_{\text{MMSINR}}$ has at least one feasible solution for the given scheduling and minimum SINR constraints. Considering this, a low-complexity sub-optimal algorithm using the frame work of CCP is developed for the MMSINR problem in the sequel.

A. Joint Design Problem Formulation: MMSINR

A tractable mathematical formulation of (3) is,

$$\begin{aligned} \mathcal{P}_1^{\text{MM}} :& \max_{\mathbf{W}} \min_{i=\{1,\dots,N\}} \{\beta_i \gamma_i\} \\ \text{s.t. } C_1 :& \left\| \left[\|\mathbf{w}_1\|_2, \dots, \|\mathbf{w}_N\|_2 \right] \right\|_0 == K, \\ C_2 :& \sum_{i=1}^{N} \|\mathbf{w}_i\|_2^2 \leq P_T, \\ C_3 :& \beta_i \gamma_i \geq \mathbf{1}\left(\|\mathbf{w}_i\|_2\right) \tilde{\epsilon}_i, \end{aligned} \tag{14}$$

where $\mathbf{1}$ is an indicator function with $\mathbf{1}\left(\|\mathbf{w}_i\|_2\right) = 0$ if $\|\mathbf{w}_i\|_2 = 0$ otherwise $\mathbf{1}\left(\|\mathbf{w}_i\|_2\right) = 1$.

The SINR γ_i is non-convex and piece-wise minimum of $\{\gamma_i\}_{i=1}^N$ is also non-convex. So, $\mathcal{P}_1^{\text{MM}}$ maximizes a non-convex objective subject to a combinatorial constraint C_1; this is generally a NP-hard problem. Moreover obtaining a global solution to $\mathcal{P}_1^{\text{MM}}$ requires an exhaustive search over all the possible sets and solving the classical MMSINR problem for each set.

Adopting classical epigraph formulation: In the classical MMSINR problem, for the predefined selected users, SINRs of all users is addressed with a slack variable, say s, that lower bounds $\beta_i\gamma_i, \ \forall i$ i.e., $\{\beta_i\gamma_i\}_{i=1}^N \geq s$ [37], [38]. However, this approach cannot be applied to the present joint design problem since there always exist $N-K$ unscheduled users whose SINR is identically zero. Therefore, lower bounding all $\{\beta_i\gamma_i\}_{i=1}^N$ with s, makes the problem trivial and the solution, say s^*, is always zero. Letting $\mathcal{S}$ to be the set of scheduled users and adopting the epigraph formulation, the problem $\mathcal{P}_1^{\text{MM}}$ is reformulated as,

$$\begin{aligned} \mathcal{P}_2^{\text{MM}} :& \max_{\mathbf{W}, s, \mathcal{S}} \ s \\ \text{s.t. } & C_1, C_2 \text{ in (14)} \end{aligned} \tag{15}$$

$$C_3 : \beta_i \gamma_i \geq s, \ \forall i \in \mathcal{S},$$

$$C_4 : s \geq \tilde{\epsilon}_i, \ \forall i \in \mathcal{S},$$

A Novel Reformulation: Similar to WSR problem, letting η_i to be a binary variable associated to user i, an equivalent formulation of $\mathcal{P}_2^{\text{MM}}$, without the set notation is,

$$\mathcal{P}_3^{\text{MM}} : \max_{\mathbf{W},\boldsymbol{\eta},s} \ s \tag{16}$$

$$\text{s.t. } C_1 : \eta_i \in \{0, 1\}, \ \forall i,$$

$$C_2 : \ \|\mathbf{w}_i\|_2^2 \leq \eta_i P_T,$$

$$C_3 : \ \sum_{i=1}^{N} \eta_i == K,$$

$$C_4 : \ \sum_{i=1}^{N} \|\mathbf{w}_i\|_2^2 \leq P_T,$$

$$C_5 : \beta_i \gamma_i \geq \eta_i \tilde{\epsilon}_i, \forall i,$$

$$C_6 : \beta_i \gamma_i \geq \eta_i s, \forall i.$$

Remarks:

- Constraint C_5 is the minimum SINR constraint equivalently written with the help of $\eta_i s$.
- The variable s in C_6 is active only when $\eta_i = 1$. For example, when user i not scheduled i.e., $\eta_i = 0$, its SINR is lower bounded by 0 which is always satisfied by the SINR definition. Similarly, when user i scheduled i.e., $\eta_i = 1$, its SINR is lower bounded by s. Thus the maximization of s optimizes the minimum SINR of only the scheduled users.

B. A Novel DC reformulation: MMSINR

The problem $\mathcal{P}_3^{\text{MM}}$ is a MINLP where the non-convexity is due to constraints C_5 and C_6, while the combinatorial nature is due to constraint C_1. Similar to constraint C_5 of $\mathcal{P}_4^{\text{WSR}}$, constraint C_5 of the problem $\mathcal{P}_3^{\text{MM}}$ can be formulated as a DC constraint. However, the same approach cannot be applicable to constraint C_6 in $\mathcal{P}_3^{\text{MM}}$ as η_i and s are both variables. Moreover, to the best of our knowledge DC reformulation of constraints of type C_6 in $\mathcal{P}_3^{\text{MM}}$ is not known. In this section, a novel procedure is proposed to transform constraints of type C_6 in $\mathcal{P}_3^{\text{MM}}$ as DC constraints.

This procedure involves the change of variable s by $\frac{1}{t}$ followed by rearrangement as described below,

$$\beta_i\gamma_i \geq \frac{\eta_i}{t} \implies 1+\beta_i\gamma_i \geq 1+\frac{\eta_i}{t} \Rightarrow \mathcal{L}_i\left(\mathbf{W},t\right)-\mathcal{H}_i\left(\mathbf{W},\eta_i,t\right) \leq 0, \tag{17}$$

where $\mathcal{I}_i\left(\mathbf{W}\right) = \sigma^2 + \sum_{j\neq i}|\mathbf{h}_i^H\mathbf{w}_j|^2$, $\mathcal{H}_i\left(\mathbf{W},\eta_i,t\right) = \frac{\mathcal{I}_i\left(\mathbf{W}\right)+\beta_i|\mathbf{h}_i^H\mathbf{w}_i|^2}{t+\eta_i}$ and $\mathcal{L}_i\left(\mathbf{W},t\right) = \frac{\mathcal{I}_i\left(\mathbf{W}\right)}{t}$. Notice that, given $t>0$, $\mathcal{L}_i\left(\mathbf{W},t\right)$ is jointly convex in $\mathbf{W}$ and t and $\mathcal{H}_i\left(\mathbf{W},\eta_i,t\right)$ is also jointly convex in $\mathbf{W},\eta_i$ and t. Hence, (17) is a DC constraint.

Letting $\mathcal{J}_i\left(\mathbf{W},\eta_i,t\right) = \frac{\mathcal{I}_i\left(\mathbf{W}\right)+\beta_i|\mathbf{h}_i^H\mathbf{w}_i|^2}{1+\eta_i\tilde{\epsilon}_i}$, for the sake of completion, with the help of variable t and (17), the problem $\mathcal{P}_3^{\text{MM}}$ is reformulated as,

$$\begin{aligned}
\mathcal{P}_4^{\text{MM}}:\ &\min_{\mathbf{W},\boldsymbol{\eta},t}\ t \qquad (18)\\
\text{s.t. } &C_1, C_2, C_3, C_4 \text{ in (16)},\\
&C_5: \mathcal{I}_i\left(\mathbf{W}\right)-\mathcal{J}_i\left(\mathbf{W},\eta_i,t\right)\leq 0, \forall i,\\
&C_6: \mathcal{L}_i\left(\mathbf{W},t\right)-\mathcal{H}_i\left(\mathbf{W},\eta_i,t\right)\leq 0,\ \forall i,\\
&C_7: t>0.
\end{aligned}$$

The problem $\mathcal{P}_4^{\text{MM}}$ is a DC problem with combinatorial constraint C_1. To circumvent the combinatorial nature, following the approach in Secetion III, the binary constraint η_i is relaxed to a box constraint between 0 and 1 and η_i is penalized with $\mathbb{P}\left(\eta_i\right)$ as,

$$\begin{aligned}
\mathcal{P}_5^{\text{MM}}:\ &\min_{\mathbf{W},\boldsymbol{\eta},t}\ t-\lambda_2\mathbb{P}\left(\eta_i\right) \qquad (19)\\
\text{s.t. } &C_1: 0\leq \eta_i \leq 1, \forall i,\\
&C_2, C_3, C_4, C_5, C_6, C_7 \text{ in (18)},
\end{aligned}$$

where $\lambda_2 \in \mathcal{R}^+$ is a penalty parameter of the design.

The problem $\mathcal{P}_5^{\text{MM}}$ maximizes a convex objective subject to convex and DC constraints. Hence $\mathcal{P}_5^{\text{MM}}$ is a DC problem and a CCP based algorithm could be solved with an FIP obtained from Section IV-D . However, the strict equality constraint C_3 in $\mathcal{P}_5^{\text{MM}}$, limits the update of the $\boldsymbol{\eta}$. In order to allow the flexibility in choosing $\boldsymbol{\eta}$, the following problem is considered instea,

$$\mathcal{P}_6^{\text{MM}}:\ \min_{\mathbf{W},\boldsymbol{\eta},t}\ t-\lambda_2\mathbb{P}\left(\eta_i\right)+\Omega\left(\sum_{i=1}^{N}\eta_i-K\right)^2 \tag{20}$$

$$\text{s.t. } C_1, C_2, C_4, C_5, C_6, C_7 \text{ in (19)},$$

where $\Omega \in \mathcal{R}^+$ is a penalty parameter. It is easy to see that choosing the appropriate Ω (usually higher value) ensures the equality constraint. The problem $\mathcal{P}_6^{\mathrm{MM}}$ is also a DC problem and a CCP based algorithm, JSP-MMSINR, is proposed in the sequel to solve it efficiently.

C. JSP-MMSINR: A Joint Design Algorithm

In this section, we propose a CCP framework based iterative algorithm to the problem $\mathcal{P}_6^{\mathrm{MM}}$, which is referred to as JSP-MMSINR, wherein the JSP-MMSINR executes the following Convexification and Optimization steps in each iteration:

- Convexification: Let $(\mathbf{W}, \boldsymbol{\eta}, t)^{k-1}$ be the estimates of $\mathbf{W}_i, \eta_i, t$ in iteration $k-1$. In iteration k, the concave part of C_5 and C_6 for user i in $\mathcal{P}_6^{\mathrm{MM}}$ i.e., $-\mathcal{H}_i(\mathbf{W}, \eta_i, t)$ and $-\mathcal{J}_i(\mathbf{W}, \eta_i, t)$ are replaced by its affine approximation around $(\mathbf{W}, \boldsymbol{\eta}, t)^{k-1}$ which is given by,

$$\tilde{\mathcal{H}}_i\left(\mathbf{W}, \boldsymbol{\eta}, t\right)^{k-1} \triangleq -\mathcal{H}_i\left(\mathbf{W}, \boldsymbol{\eta}, t\right)^{k-1} - \mathbb{R}\left\{\nabla^H \mathcal{H}_i\left(\mathbf{W}, \boldsymbol{\eta}, t\right)^{k-1} \begin{bmatrix} \mathbf{w}_1 - \mathbf{w}_1^{k-1} \\ \vdots \\ \mathbf{w}_N - \mathbf{w}_N^{k-1} \\ \eta_i - \eta_i^{k-1} \\ t - t^{k-1} \end{bmatrix}\right\},$$

$$\tilde{\mathcal{J}}_i\left(\mathbf{W}, \boldsymbol{\eta}, t\right)^{k-1} \triangleq -\mathcal{J}_i\left(\mathbf{W}, \boldsymbol{\eta}, t\right)^{k-1} - \mathbb{R}\left\{\nabla^H \mathcal{J}_i\left(\mathbf{W}, \boldsymbol{\eta}, t\right)^{k-1} \begin{bmatrix} \mathbf{w}_1 - \mathbf{w}_1^{k-1} \\ \vdots \\ \mathbf{w}_N - \mathbf{w}_N^{k-1} \\ \eta_i - \eta_i^{k-1} \\ t - t^{k-1} \end{bmatrix}\right\}, \quad (21)$$

where $\nabla \mathcal{H}_i\left(\mathbf{W}, \boldsymbol{\eta}, t\right)^{k-1}$ and $\nabla \mathcal{J}_i\left(\mathbf{W}, \boldsymbol{\eta}, t\right)^{k-1}$ are the evaluated gradients of $\mathcal{H}_i\left(\mathbf{W}, \boldsymbol{\eta}, t\right)$ and $\mathcal{J}_i\left(\mathbf{W}, \boldsymbol{\eta}, t\right)$ at $(\mathbf{W}, \boldsymbol{\eta}, t)^{k-1}$ respectively. The expressions for $\nabla \mathcal{H}_i\left(\mathbf{W}, \boldsymbol{\eta}, t\right)^{k-1}$ and $\nabla \mathcal{J}_i\left(\mathbf{W}, \boldsymbol{\eta}, t\right)^{k-1}$ can be obtained by following (11). Similarly, the first order Taylor series approximation of the objective in $\mathcal{P}_6^{\mathrm{MM}}$ after ignoring the constant terms,

$$\mathcal{F}\left(t, \boldsymbol{\eta}\right) = t - \lambda_2 \sum_{i=1}^{N} \eta_i \nabla \mathbb{P}\left(\eta_i^{k-1}\right) + \Omega\left(\sum_{i=1}^{N} \eta_i - K\right)^2$$

- Optimization: The update $(\mathbf{W}, \boldsymbol{\eta}, t)^k$ is obtained by solving the following convex problem:

$$\begin{aligned}
&\mathcal{P}_7^{\text{MM}} : \max_{\mathbf{W},\boldsymbol{\eta},t} \mathcal{F}(t, \boldsymbol{\eta}) \\
&\text{s.t. } C_1, C_2, C_3, C_4 \text{ in (20)} \\
&\quad C_5 : \mathcal{I}_i(\mathbf{W}, t) + \tilde{\mathcal{J}}(\mathbf{W}, \boldsymbol{\eta}, t)^{k-1} \leq 0, \ \forall i, \\
&\quad C_6 : \mathcal{L}_i(\mathbf{W}, t) + \tilde{\mathcal{H}}(\mathbf{W}, \boldsymbol{\eta}, t)^{k-1} \leq 0, \ \forall i.
\end{aligned}$$

D. Feasible Initial Point: MM-SINR

Notice that JSP-MMSINR is a CCP framework based algorithm and hence a FIP is sufficient for the algorithm to converge to a stationary point [27]. Unlike WSR problem, obtaining a trivial FIP to the problem $\mathcal{P}_6^{\text{MM}}$ is difficult as initializing $\mathbf{W}$ to all zeros results in zero SINR for all the users and thus $t = 0$ where later is the violation of the constraint C_5. However, one may find a FIP by the following iterative procedure.

- Step 1: Initialize $\boldsymbol{\eta} = \hat{\boldsymbol{\eta}}$ that satisfies constraints C_1 and C_3 in $\mathcal{P}_6^{\text{MM}}$.
- Step 2: Solve the following optimization:

$$\begin{aligned}
&\mathcal{P}_{\text{FESMM}} : \{\hat{\mathbf{W}}\} : \ \text{find } \mathbf{W} \\
&\text{s.t } \tilde{C}_1 : \left\| \left[\sigma \ldots \{\mathbf{h}_i^H \mathbf{w}_j\}_{j \neq i} \ldots \right] \right\|_2 \leq \mathbf{h}_i^H \mathbf{w}_i \sqrt{\frac{\beta_i}{\eta_i \tilde{\epsilon}_i}}, \ \forall i, \\
&\quad \tilde{C}_2 : \mathbb{R}\{\mathbf{h}_i^H \mathbf{w}_i\} \geq 0, \forall i, \\
&\quad \tilde{C}_3 : \Im\{\mathbf{h}_i^H \mathbf{w}_i\} == 0, \forall i, \\
&\quad \tilde{C}_4 : \sum_{i=1}^{N} \|\mathbf{w}_i\|_2^2 \leq P_T,
\end{aligned} \tag{22}$$

  where FESMM is a acronym used for feasibility problem of MMSINR.
- Step 3: Exit the loop if $\hat{\mathbf{W}}$ from step 2 is feasible and $t^0 = \dfrac{1}{\min_i\{\eta_i \tilde{\epsilon}_i\}}$ else set $\boldsymbol{\eta} = \delta \hat{\boldsymbol{\eta}}$ and continue to step 2.

Remarks:

- By construction, the initial $\hat{\boldsymbol{\eta}}$ from step 1 is always feasible to $\mathcal{P}_6^{\text{MM}}$.
- Efficient algorithms to solve the convex precoding problem in step 2 is proposed in [6] and [7], and is solvable globally using tools like CVX.

- The number of iterations that are needed to obtain a FIP from above procedure depends on $\hat{\boldsymbol{\eta}}$, δ and K. Suppose, if the initial $\hat{\boldsymbol{\eta}} \approx 0$, a FIP is obtained in one iteration with high probability. Similarly, if the initial $\hat{\boldsymbol{\eta}} \approx 1$ the solution from above can be infeasible in the initial iterations. For the latter case, smaller δ leads to a FIP in few iterations and larger δ takes longer iterations to find a FIP.

Notice that a FIP obtained from this process is only feasible to problem $\mathcal{P}_6^{\text{MM}}$ but not to the problem $\mathcal{P}_4^{\text{MM}}$ since it violates scheduling constraint and binary constraint of $\boldsymbol{\eta}$. However, appropriate adaptation of the penalty parameters λ_2 and Ω (e.g. monotonic increment) ensures that the obtained final solution from algorithm 2 is always feasible to $\mathcal{P}_4^{\text{MM}}$.

Letting $\mathcal{P}_7^{\text{MM}}(k)$ be the objective value of the problem $\mathcal{P}_7^{\text{MM}}$ at iteration k, the pseudocode of JSP-MMSINR for the joint design problem is given in algorithm 2.

Algorithm 2 JSP-MMSINR

Input: $\mathbf{H}, [\tilde{\epsilon}_1, \ldots, \tilde{\epsilon}_N], P_T, \Delta$, $\boldsymbol{\eta}^0$, $\mathbf{W}^0$, $\lambda_1 = 0$, $k = 1$

Output: $\mathbf{W}, \boldsymbol{\eta}$, t

while $|\mathcal{P}_7^{\text{MM}}(k) - \mathcal{P}_7^{\text{MM}}(k-1)| \geq \Delta$ **do**

  Convexification: Convexify the problem (21)

  Optimization: Update $(\mathbf{W}, \boldsymbol{\eta}, t)^k$ by solving $\mathcal{P}_7^{\text{MM}}$

  Update : $\mathcal{P}_7^{\text{MM}}(k), \lambda_2, k$;

end while

E. Complexity: MM-SINR

Similar to JSP-WSR, the computational complexity of JSP-MMSINR depends on the complexities of iterative procedures proposed in Section IV-C and Section IV-D. The proposed JSP-MMSINR in Section III-C is a CCP based iterative algorithm; hence, the complexity of the algorithm depends on complexity of the sub-problems $\mathcal{P}_7^{\text{MM}}$. The problem $\mathcal{P}_7^{\text{MM}}$ has $(NM + N + 1)$ decision variables, $2N + 1$ convex and $2N + 1$ linear constraints, hence the computational complexity of $\mathcal{P}_7^{\text{MM}}$ is $\mathcal{O}\left((NM + N + 1)^3 (4N + 2)\right)$. Similarly, the computational complexity of the procedure in Section IV-D depends on the per iteration complexity of $\mathcal{P}_{\text{FESMM}}$. $\mathcal{P}_{\text{FESMM}}$

is a convex problem with MN decision variables, $2N+1$ convex constraints and $2N$ linear constraints. Hence, the computational complexity of $\mathcal{P}_{\text{FESMM}}$ is $\mathcal{O}\left((MN)^3\,(4N+1)\right)$ [36].

V. Power Minimization

In this section, we consider the joint design problem with the objective of minimizing the sum power consumed at the BS subject to scheduling of K users whose minimum SINR requirement is met. As mentioned previously, scheduling utmost K users leads to the trivial solution of no users being scheduled which results in zero consumed power.

A. Joint Design Problem Formulation: PMIN

Similar to Section IV, the user scheduling is handled through the norm of the precoder as shown in (5). With the help of (5) and notations defined, and letting $\bar{\mathcal{S}}$ to be the set of scheduled users, a tractable formulation of $\mathcal{P}_{\text{PMIN}}$ solely as a function of precoding vectors as follows:

$$\begin{aligned}\mathcal{P}_1^{\text{PMIN}}:\ &\min_{\mathbf{W},\bar{\mathcal{S}}}\ \sum_{i\in\bar{\mathcal{S}}}\|\mathbf{W}_i\|_2^2 \\ \text{s.t. } C_1:\ &\left\|\left[\|\mathbf{w}_1\|_2,\ldots,\|\mathbf{w}_N\|_2\right]\right\|_0 == K, \\ C_2:\ &\gamma_i\geq\tilde{\epsilon}_i,\ i\in\bar{\mathcal{S}}.\end{aligned} \tag{23}$$

The problem $\mathcal{P}_1^{\text{PMIN}}$ is combinatorial due to the constraints C_1 and C_2 and also non-convex due to $\{\gamma_i\}_{i=1}^N$ in constraint C_2. Letting $\Upsilon\in\mathcal{R}^+$ to be a constant, a mathematically tractable formulation that allows us to design a low-complexity algorithm is

$$\begin{aligned}\mathcal{P}_2^{\text{PMIN}}:\ &\min_{\mathbf{W},\boldsymbol{\eta}}\ \|\mathbf{W}\|_2^2 \\ \text{s.t. } C_1:\ &\eta_i\in\{0,1\},\ \forall i, \\ C_2:\ &\|\mathbf{w}_i\|_2^2\leq\eta_i\Upsilon,\ \forall i, \\ C_3:\ &\sum_{i=1}^N\eta_i == K, \\ C_4:\ &\gamma_i\geq\tilde{\epsilon}_i\eta_i,\ \forall i.\end{aligned} \tag{24}$$

Remarks:

- For $\eta_i=1$, Υ in C_2 provides upper bound on the power of user i. Moreover, a lower bound on Υ would be the total system power.

A DC reformulation: The problem $\mathcal{P}_2^{\text{PMIN}}$ is an MINLP due to combinatorial constraint C_1 and non-convex constraint C_4. Similar to WSR and MMSINR problems, using the DC formulation of constraint C_4 and penalization method for C_1, the DC formulation of the problem $\mathcal{P}_2^{\text{PMIN}}$ is,

$$\mathcal{P}_3^{\text{PMIN}}: \min_{\mathbf{W},\boldsymbol{\eta}} \ \|\mathbf{W}\|_2^2 - \lambda_3 \sum_{i=1}^{N} \mathbb{P}(\eta_i) \tag{25}$$

$$\text{s.t. } C_1: 0 \leq \eta_i \leq 1, \ \forall i,$$

$$C_2, C_3 \text{ in (24)},$$

$$C_4: \mathcal{I}_i(\mathbf{W}) - f_i(\mathbf{W}, \eta_i), \ \forall i,$$

where $\lambda_3 \in \mathcal{R}^+$ is the penalty parameter and $f_i(\mathbf{W}, \eta_i) = \dfrac{\mathcal{I}_i(\mathbf{W}) + |\mathbf{h}_i^H \mathbf{w}_i|^2}{1 + \tilde{\epsilon}_i \eta_i}$.

The problem $\mathcal{P}_3^{\text{PMIN}}$ is a DC problem which can be solved using CCP. However, finding a FIP becomes difficult as for chosen $\boldsymbol{\eta}$, $\mathcal{P}_3^{\text{PMIN}}$ may become infeasible [6]. For the ease of finding an FIP, the constraint C_2 in $\mathcal{P}_4^{\text{PMIN}}$ is relaxed and penalized as follows:

$$\mathcal{P}_4^{\text{PMIN}}: \min_{\mathbf{W},\boldsymbol{\eta}} \|\mathbf{W}\|_2^2 - \Omega \sum_{i=1}^{N} \mathbb{P}(\eta_i) + \mu \left(\sum_{i=1}^{N} \eta_i - K \right)^2 \tag{26}$$

$$\text{s.t. } C_1, C_2, C_4 \text{ in (25)}$$

where $\mu > 0$ is penalty parameter. Notice that for the appropriate μ, equality constraint is ensured. Moreover, The problem $\mathcal{P}_4^{\text{PMIN}}$ is a DC problem which solvable using CCP.

B. Joint Design Algorithm: PMIN

In this section, following the CCP framework proposed in Section IV-C, the CCP based algorithm for PMIN is proposed. The proposed joint scheduling and precoding (JSP) for PMIN (JSP-PMIN) algorithm executes the following two steps iteratively until the convergence:

- Convexification: Let $(\mathbf{W}, \boldsymbol{\eta})^{k-1}$ be the estimates of $(\mathbf{W}, \boldsymbol{\eta})$ in iteration $k-1$. In iteration k, the concave part of C_3 in $\mathcal{P}_4^{\text{PMIN}}$ for user i i.e., $-f_i(\mathbf{W}, \eta_i)$ is replaced by its affine approximation around the estimate of $(\mathbf{W}, \boldsymbol{\eta})^{k-1}$ which is given by,

$$\tilde{f}(\mathbf{W}, \eta_i) \triangleq -f(\mathbf{W}, \eta_i)^{k-1} - \mathbb{R}\left\{ \nabla^H f(\mathbf{W}, \eta_i)^{k-1} \begin{bmatrix} \mathbf{w}_1 - \mathbf{w}_1^{k-1} \\ \vdots \\ \mathbf{w}_N - \mathbf{w}_N^{k-1} \\ \eta_i - \eta_i^{k-1} \end{bmatrix} \right\}. \tag{27}$$

- Optimization: Update $(\mathbf{W},\boldsymbol{\eta})^k$ is obtained by solving the following convex problem:

$$\mathcal{P}_5^{\text{PMIN}}:\min_{\mathbf{W},\boldsymbol{\eta}} \|\mathbf{W}\|_2^2+\mu\left(\sum_{i=1}^{N}\eta_i-K\right)^2-\lambda_3\sum_{i=1}^{N}\eta_i\nabla\mathbb{P}\left(\eta_i^{k-1}\right) \tag{28}$$
$$\text{s.t. } C_1: 0\le\eta_i\le 1,\ \forall i,$$
$$C_2:\ \|\mathbf{w}_i\|_2^2\le\eta_i\Upsilon,\forall i,$$
$$C_3:\mathcal{I}_i\left(\mathbf{W}\right)+\tilde{f}(\mathbf{W}_i,\eta_i)^{k-1}\le 0,\ \forall i.$$

The convex problem $\mathcal{P}_5^{\text{PMIN}}$ has $(NM+N)$ decision variables, $2N$ convex and $2N$ linear constraints, hence the computational complexity of $\mathcal{P}_5^{\text{MM}}$ is $\mathcal{O}\left((NM+N)^3\,(4N)\right)$.

C. Feasible Initial Point: PMIN

An initial feasible point, which suffices the convergence of JSP-PMIN to a stationary point [27], for the problem $\mathcal{P}_5^{\text{PMIN}}$ is obtained by the following iterative procedure.

- Step 1: Initialize $\boldsymbol{\eta}=\hat{\boldsymbol{\eta}}$ that satisfies C_1 and C_3 in $\mathcal{P}_4^{\text{PMIN}}$.
- Step 2: Solve the following optimization:

$$\mathcal{P}_{\text{FESPMIN}}:\{\hat{\mathbf{W}}\}:\ \text{find } \mathbf{W} \tag{29}$$
$$\text{s.t } \tilde{C}_1:\left\|\left[\sigma\ldots\{\mathbf{h}_i^H\mathbf{w}_j\}_{j\neq i}\ldots\right]\right\|_2\le\frac{\mathbf{h}_i^H\mathbf{w}_i}{\sqrt{\eta_i\bar{\tilde{\epsilon}}_i}},\ \forall i,$$
$$\tilde{C}_2:\mathbb{R}\{\mathbf{h}_i^H\mathbf{w}_i\}\ge 0,\forall i,$$
$$\tilde{C}_3:\Im\{\mathbf{h}_i^H\mathbf{w}_i\}==0,\forall i.$$
$$\tilde{C}_4:\|\mathbf{W}\|_2^2\le P_T$$

- Step 3: Exit the loop if $\hat{\mathbf{W}}$ is feasible (see [6]) else set $\boldsymbol{\eta}=\delta\hat{\boldsymbol{\eta}}$ and continue to step 2.

Notice that a FIP obtained from the above procedure may not be feasible to $\mathcal{P}_2^{\text{PMIN}}$ since it may violate binary and scheduling constraints. However, the adopted penalty methods ensure the scheduling and binary constraints. Hence, the final solution obtained from 3 is always a feasible solution to $\mathcal{P}_2^{\text{PMIN}}$.

Letting $\mathcal{P}_5^{\text{PMIN}}(k)$ be the objective value of the problem $\mathcal{P}_5^{\text{PMIN}}$ at iteration k, The pseudo code of the algorithm is illustrated in the algorithm 3.

Algorithm 3 JSP-PMIN

Input: $\mathbf{H}$, $\left[\bar{\bar{\epsilon}}_1, \ldots, \bar{\bar{\epsilon}}_N\right]$, Δ, $\boldsymbol{\eta}^0$, $\mathbf{W}^0$, $\lambda_1 = 0,\ k = 1$

Output:$\mathbf{W}, \boldsymbol{\eta}$

while $|\mathcal{P}_6^{\text{PMIN}}(k) - \mathcal{P}_6^{\text{PMIN}}(k-1)| \geq \Delta$ **do**

  Convexification: Convexify the problem (21)

  Optimization: Update $\left(\mathbf{W}^k, \boldsymbol{\eta}^k\right)$ by solving $\mathcal{P}_6^{\text{PMIN}}$

  Update : $\mathcal{P}_6^{\text{PMIN}}(k), \Omega, k$

end while

VI. Simulation results

A. Simulation Setup

In this section, we evaluate the performance of the proposed algorithms for the MMSINR, WSR and PMIN problems. The system parameters and benchmark scheduling method discussed in this paragraph are common for all the figures. Entries of the channel matrix, i.e., $\{h_{ij}\}$s are drawn from the complex normal distribution with zero mean and unit variance and noise variances are considered to be unity i.e., $\sigma^2 = 1$. Simulation results in all the figures are averaged over 500 different channel realizations (CRs). The penalty parameter λ_1 is initialized to 0.5 and incremented as $\lambda_1 = 1.1\lambda_1$ until $\lambda_1 \leq 10$. For all the simulations of MMSINR and PMIN, K is chosen as M. By the nature of MMSINR (PMIN) design, dropping the user with the lowest SINR (higher power) leads to a better objective. This phenomenon continues until it drops $N - M$ users and can not drop any further due to the scheduling constraint. Since, this naturally enforces the binary nature of $\boldsymbol{\eta}$, $\lambda_2 = 0$ ($\lambda_3 = 0$) in MMSINR (PMIN) still yields the binary $\boldsymbol{\eta}$ which is shown Section VI-D and VI-E. Hence, λ_2 and λ_3 are fixed zero in all iterations. The penalty parameters Ω and μ are initialized to 0.01 and incremented as $\Omega = 1.2\Omega$ and $\mu = 1.2\mu$ in each iteration until $\Omega \leq 20$ and $\mu \leq 20$.

B. Benchmark algorithms

To evaluate the performance of the proposed JSP algorithms - due to the lack of a comparable joint solution - the following benchmarks (iterative decoupled solutions that execute the following steps in sequence) are devised:

- In step 1, users are scheduled according to proposed weighted semi-orthogonal user scheduling (WSUS) or exhaustive search-based user scheduling (ES) or random user scheduling (RUS). The considered WSUS is an extension of the SUS algorithm proposed in [9]. In SUS, the users are selected sequentially based on the orthogonality of their channels with those of already scheduled ones. In WSUS, orthogonality indices calculated according to SUS are multiplied with their associated weights and the user with the highest weighted orthogonality index is scheduled. This process is repeated until M users are scheduled.
- In step 2, the precoding problem for the scheduled users is solved by the following methods:
  - It is easy to see that, retaining only the terms corresponding to scheduled users by substituting corresponding η_is to 1 (rest are made zero) and ignoring the constraint solely dependent on η_is in (9), (20) gives the DC formulation of the precoding problem for the scheduled users for WSR and MMSINR and respectively. These precoding problems can be solved using CCP with a FIP obtained from $\mathcal{P}_{\text{FESWSR}}$ and $\mathcal{P}_{\text{FESPMIN}}$ by substituting corresponding η_is with 1. RUS, ES, SUS and WSUS combined with this proposed WSR is simply referred to as RUS-WSR, ES-WSR, SUS-WSR, and WSUS-WSR respectively and for MMSINR as RUS-MMSINR, ES-MMSINR, SUS-MMSINR, and WSUS-MMSINR respectively. Similarly, RUS, ES, SUS and WSUS based scheduling followed by the SDP based power minimization proposed in [7] is used for PMIN precoding problem and is referred to simply as RUS-PMIN, ES-PMIN, SUS-PMIN, and WSUS-PMIN respectively.
  - An SDR version of DC formulation proposed in [24] also used for solving the precoding for the scheduled users in WSR case as a reference hence is referred to as RWSR. WSUS combined with RWSR is referred to as WSUS-RWSR.
- In step 3: If the precoding problem in step 2 is infeasible exit the loop else drop the user with least orthogonality and repeat step 2 for an updated set of scheduled users. However, the precoding problems for MMSINR and PMIN are assumed to be feasible.

C. WSR Performance Evaluation

In figure 1a, we compare the performance of JSP-WSR as a function of N varying from 15 to 30 in steps of 5 for $M = 10$, $P_T = 10$dB and $\tilde{\epsilon}_i = 4$dB, $\forall i$. Weights $\{\alpha_i\}_{i=1}^{N}$ are randomly drawn from the set $\{\frac{k}{N}\}, k = 1, \ldots, N$. In figure 1a, RUS-WSR, SUS-WSR, WSUS-WSR and WSUS-RWSR are the decoupled benchmark algorithms. The JSP-WSR initialized with a trivial

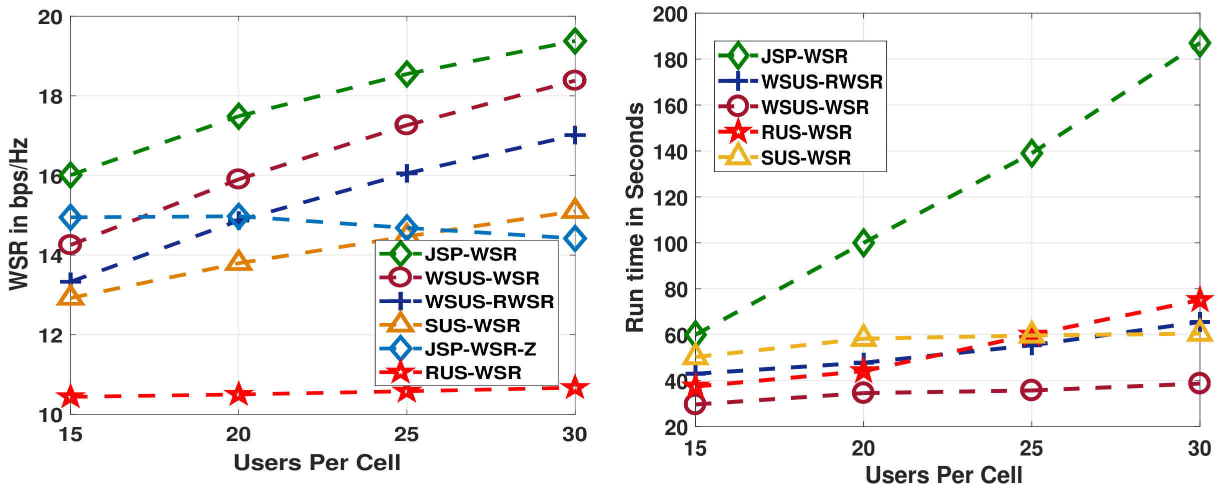


Fig. 1: Comparison of different WSR optimization approaches for M =10, $\{\tilde{\epsilon}_i = 4\text{dB}\}_{i=1}^{N}$, $P_T = 10$ dB, and N is varied from 15 to 30 (a) Achieved WSR and (b) algorithm run time

solution ($\mathbf{W}^0 = \mathbf{0}$, $\boldsymbol{\eta}^0 = \mathbf{0}$) is referred to as JSP-WSR-Z and JSP-WSR initialized with an FIP obtained from Section III-D continues to be referred to as JSP-WSR.

From figure 1a, it is clear that the joint solution JSP-WSR outperforms all the other decoupled benchmarks. Although JSP-WSR, RUS-WSR, SUS-WSR, and WSUS-WSR have the same underlying precoding algorithm, JSP-WSR achieves better performance as it jointly updates scheduling and precoding. Considering weights into scheduling in WSUS-WSR improves over SUS-WSR, as shown in figure 1a, but it still outperformed by JSP-WSR. However, the gains diminish as N increases as the probability of finding nearly orthogonal user channels (for the considered Gaussian model) increases; this implies that the user scheduling has minimum impact on performance. Hence, WSUS-WSR performs close to JSP-WSR for N relatively larger than M. However, the gains obtained by JSP-WSR even in comparison with WSUS-WSR still amounts up to 28% ($N = 15$). Notice that despite the difference in the rate of growth, all methods benefit from multiuser diversity to improve SR as N increases.

Notice that JSP-WSR and JSP-WSR-Z are identical except the FIPs. JSP-WSR and JSP-WSR-Z are CCP based algorithms hence the performance differentiation depends on FIP. Figure 1a shows that while a poor FIP like $\mathbf{W}^0 = \mathbf{0}$, $\boldsymbol{\eta}^0 = \mathbf{0}$ results in worse performance than decoupled solutions, the FIPs from Section III-D achieves better performance. This shows the efficiency of

TABLE I: Convergence rate of JSP-WSR for $M = 10$, $\{\tilde{\epsilon}_i = 4\text{dB}\}_{i=1}^{N}$, $P_T = 10$ dB as a function of N.

Number of users in a cell	Average number of iterations to converge
$N = 15$	16
$N = 20$	20.3
$N = 25$	22.5
$N = 30$	24.8

the FIP mechanism detailed in Section III-D. In particular, $\mathbf{W}^0 = \mathbf{0}$, $\boldsymbol{\eta}^0 = \mathbf{0}$ is a bad choice since it is the solution that achieves lowest WSR i.e., zero and hence the solutions of JSP-WSR-Z are generally the stationary points around the lowest objective.

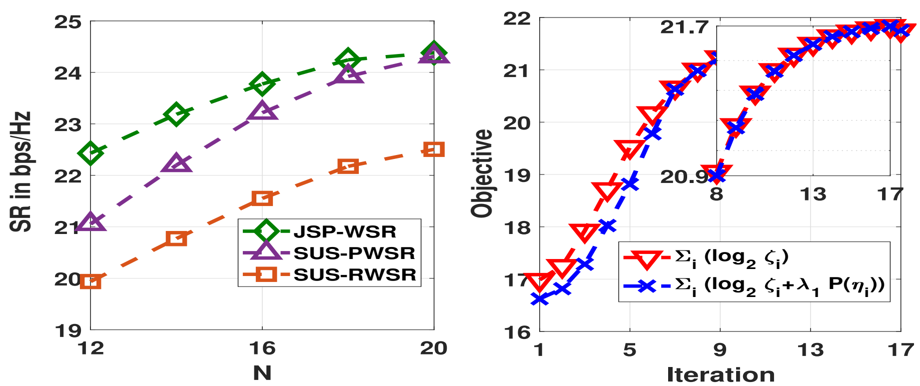
Despite having the same WSUS scheduling algorithm and the same FIP for precoding, WSUS-WSR outperforms WSUS-RWSR due to the difference in precoding algorithms as shown in figure 1a. Although classical WSR can be formulated as a DC problem using proposed reformulations and also by the approach in [24], due to the efficiency of proposed reformulations, WSUS-WSR achieves the better objective which is confirmed by figure 1a.

Figure 1b illustrates the complexity of algorithms as a function of running time in seconds. Notice that the running time includes the time to calculate the FIPs and the final solutions. In the decoupled algorithms i.e., WSUS-WSR and WSUS-RWSR the complexity of scheduling algorithms is negligible compared to the latter precoding problem. Since the precoding is always performed on M users, the precoding complexity of RUS-WSR, WSUS-WSR, and WSUS-RWSR is only a function of M. On the contrary, joint design algorithms, JSP-WSR, JSP-WSR-Z operate on N users hence the complexity increases with N. However, due to the efficiency in the design of JSP-WSR, its complexity can be comparable to that of WSUS-WSR for relatively low values of N, e.g. $N = 15$.

In table II, the performance of JSP-WSR is compared with ES-WSR and WSUS-WSR for $M = 3$, $\{\tilde{\epsilon}_i = 4\text{dB}\}_{i=1}^{N}$, $P_T = 10$ dB, and N is varied from 5 to 7 in steps of 1. Although the JSP-WSR is guaranteed to converge only to a stationary point theoretically, the results in table II confirms that these stationary points are indeed high-quality solutions. On the other hand, the shortcomings of the decoupled solution i.e., WSUS-WSR, leads to a large performance gap from both ES-WSR and JSP-WSR.

TABLE II: Performance comparison of JSP-WSR with ES-WSR for M =3, $\{\tilde{\epsilon}_i = 4\text{dB}\}_{i=1}^{N}$, P_T = 10 dB, and N is varied from 5 to 7 insteps of 1.

Number of users in a cell (N)	weighted sum rate in bps/Hz		
	ES-WSR	JSP-WSR	WSUS-WSR
$N = 5$	5.83	5.67	5.32
$N = 6$	6.51	5.97	5.59
$N = 7$	6.64	6.33	6.02



(a) (b)

Fig. 2: Comparison of different WSR optimization approaches for uniform weighted case with M =10, $\{\tilde{\epsilon}_i = 4\text{dB}\}_{i=1}^{N}$, $P_T = 10$dB (a) N varying from 12 to 20 in steps of 2. (b) convergence of the JSP-WSR (with penalty) and convergence of $\boldsymbol{\eta}$ to binary for $N = 20$.

The performance of the JSP-WSR is illustrated for uniform weighted case i.e. $\{\alpha_i = 1\}_{i=1}^{N}$ in figure 2a as a function of N. The performance gain by jointly updating scheduling and precoding in JSP-WSR over the decoupled SUS-WSR and SUS-RWSR is clear from figure 2a. However, as N increases ($N \approx 20$), SUS schedules the users with strong channel gains and least interference; hence SUS-WSR performs close to JSP-WSR. Despite the efficiency of SUS in the region around $N = 20$, SUS-RWSR performs poor due to the inefficiency of the RWSR precoding scheme.

Figure 2b illustrates the convergence behavior of the JSP-WSR and the convergence of $\boldsymbol{\eta}$ to binary values as a function of iterations. The SR obtained in each iteration is shown by the red curve while the penalized SR is shown by the blue curve. As the FIP of JSP-WSR contains a

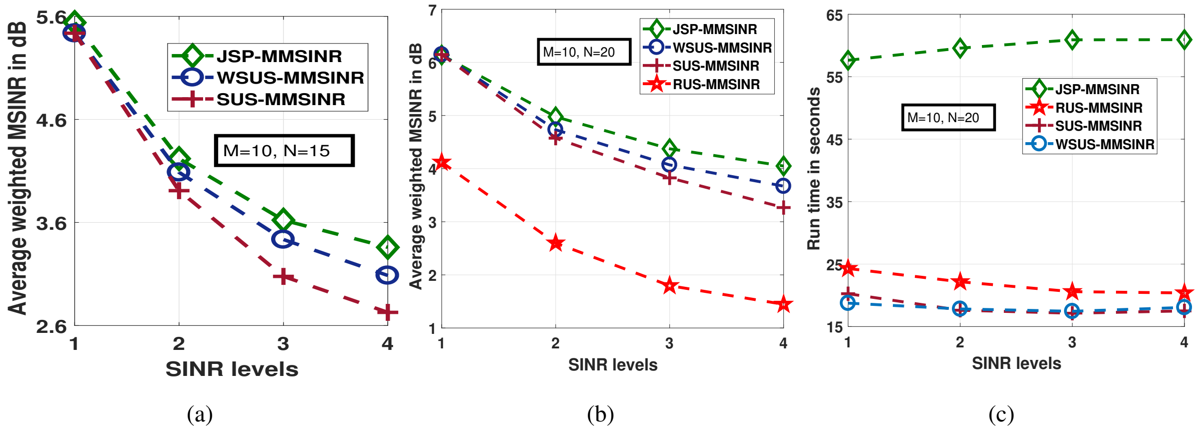


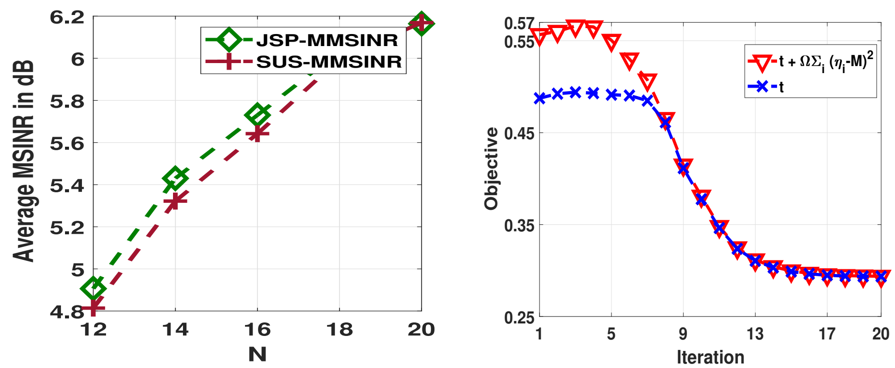
Fig. 3: Comparison of different MMSINR optimization approaches for $P_T = 10$dB, $\{\tilde{\epsilon}_i = 0\text{dB}\}_{i=1}^{N}$ and SINR levels are varied from 1 to 4 (a) $N = 15$ (b) $N = 20$ (C) algorithm run time.

non-binary $\boldsymbol{\eta}$, the solutions obtained in the initial iterations include the non-binary $\boldsymbol{\eta}$; hence, the difference between SR (red curve) and SR plus penalty (blue curve). However, as the penalty factor (λ_1) increases over the iterations, JSP-WSR favors the solutions with binary η_is. As a result, the penalty approaches zero over the iterations i.e., $\mathbb{P}(\eta_i) \approx 0, \forall i$. This behavior is clear from iteration 8 onwards. Moreover, the convergence behavior of the JSP-WSR to a stationary point of $\mathcal{P}_5^{\text{WSR}}$ is shown by the convergence of the blue curve which depicts its objective value.

D. *MMSINR Performance Evaluation*

Figure 3 illustrates the weighted minimum SINR (MSINR) of the scheduled users (averaged over 500 different CRs and referred to as average weighted MSINR) as a function of SINR levels. For SINR level 1, 2, 3 and 4, the weight β_i associated with user i is randomly drawn from the sets $\{1\}$, $\{0.5, 1\}$, $\{0.333, 0.6666, 0.9999\}$ and $\{0.25, 0.5, 0.75, 1\}$ respectively. For example, for SINR levels 2, β_i is randomly selected from $\{0.5, 1\}$. Hence the MMSINR requirement of each user is $\tilde{\epsilon}_i/0.5$ or $\tilde{\epsilon}_i$ (also $\tilde{\epsilon}_i = 1$). Notice that a higher value of β_i increases the likeliness of user i being scheduled.

The performance of JSP-MMSINR is compared with SUS-MMSINR and WSUS-MMSINR for $M = 10$, $\{\tilde{\epsilon}_i = 1 \text{ (0 dB) }\}_{i=1}^{N}$, $P_T = 10$dB and $N = 15$ in figure 3a and $N = 20$ in figure 3b. It is clear from figure 3a and 3b , that the joint solution JSP-MMSINR improves the performance over the decoupled design RUS-MMSINR, SUS-MMSINR, and WSUS-MMSINR. Despite identical



(a) (b)

Fig. 4: Comparison of different MMSINR optimization approaches for uniform weighted case with M =10,$\{\beta_i = 1, \tilde{\epsilon}_i =$ 0dB$\}_{i=1}^{N}$, $P_T = 10$ dB (a) N varying from 12 to 20 in steps of 2. (b) Convergence of the JSP-MMSINR (with penalty) and convergence of $\boldsymbol{\eta}$ to binary for $N = 20$.

underlying precoding scheme in JSP-MMSINR, RUS-MMSINR, SUS-MMSINR, and WSUS-MMSINR, the systematic approach of joint scheduling and precoder update considering the weights helps JSP-MMSINR to achieve better performance. The naive user scheduling based method i.e., RUS-MMSINR clearly performs poorer than other benchmark methods. Although WSUS-MMSINR achieves better performance over SUS-MMSINR by considering the weights into scheduling, it still performs worse than JSP-MMSINR showing the inefficiency of decoupled design. The gains obtained by JSP-MMSINR compared to best performing decoupled method i.e., WSUS-MMSINR amounts up to 10% (figure 3b, SINR level 4).

In figure 3c, the run time of the algorithms is illustrated as a function of SINR levels for $M = 10$, $N = 20$ and $P_T = 10$dB. Figure 3c shows that the gains of JSP-MMSINR are achieved at the expense of high computational complexity as illustrated in figure 3c. Moreover, the complexity of JSP-MMSINR increases as SINR levels increase. The increase in SINR levels enforces the inclusion of users with higher SINR requirement since the users with lower SINR requirement may not be sufficient to schedule exactly M users. Hence, JSP-MMSINR takes relatively longer time to converge compared lower SINR levels.

The performance of JSP-MMSINR is illustrated for uniform weighted case i.e., $\{\beta_i = 1\}_{i=1}^{N}$ in figure 4 for $M = 10$ and $P_T = 10$dB. In figure 4a, the average MSINR is illustrated as a function of N varying from 12 to 18 in steps of 2. The superior performance of JSP-MMSINR

TABLE III: Performance comparison of JSP-MMSINR with ES-MMSINR and WSUS-MMSINR for M =3, SINR level 4, $P_T = 10$dB and N is varied from 5 to 7 insteps of 1.

Number of users in a cell (N)	Average MSINR in dB with		
	ES-MMSINR	JSP-MMSINR	WSUS-MMSINR
$N = 5$	5.23	5.15	4.63
$N = 6$	5.95	5.79	5.03
$N = 7$	6.71	6.56	5.99

over SUS-MMSINR is clear from 4a. However, the gains diminish as N increases as the SUS based solution becomes efficient as mentioned previously.

In figure 4b, the convergence behavior of the algorithm and progression towards achieving exact scheduling constraint i.e., $\sum_{i=1}^{N} \eta_i == M$ are illustrated as a function of the iteration number. While the blue curve depicts the inverse of MSINR (i.e., t in $\mathcal{P}_6^{\text{MM}}$) achieved over the iteration, the red curve depicts the penalized objective where the penalty aims to satisfy the constraint of scheduling exactly M users. As FIPs violate the exact scheduling constraint, the penalized objective (red curve) is far from the objective (blue curve). However, increasing the penalty parameter Ω over the iterations until $\Omega \leq 20$ ensures the scheduling constraint. This behavior is observed from iteration 8 in figure 4b as the difference between penalized objective and objective is approximately zero. Moreover, the binary nature of $\boldsymbol{\eta}$ is also achieved over the iterations due to nature of MMSINR for fixed $\lambda_2 = 0$ in figure 3 and 4.

Table III compares the performance of JSP-MMSINR, WSUS-MMSINR and ES-WSR for $M = 3$, SINR level 4, $P_T = 10$ dB. The relatively similar performance of JSP-MMSINR and ES-MMSINR confirms the efficiency of JSP-MMSINR and high-quality nature of stationary points that the JSP-MMSINR converges to.

E. PMIN Performance Evaluation

The total power consumed by the scheduled users (for each channel realization) is averaged over 500 channel realizations (CRs) which is referred to as average total power per CR. In figure 5, the average total power per CR is depicted as a function of SINR levels for $M = 10$,

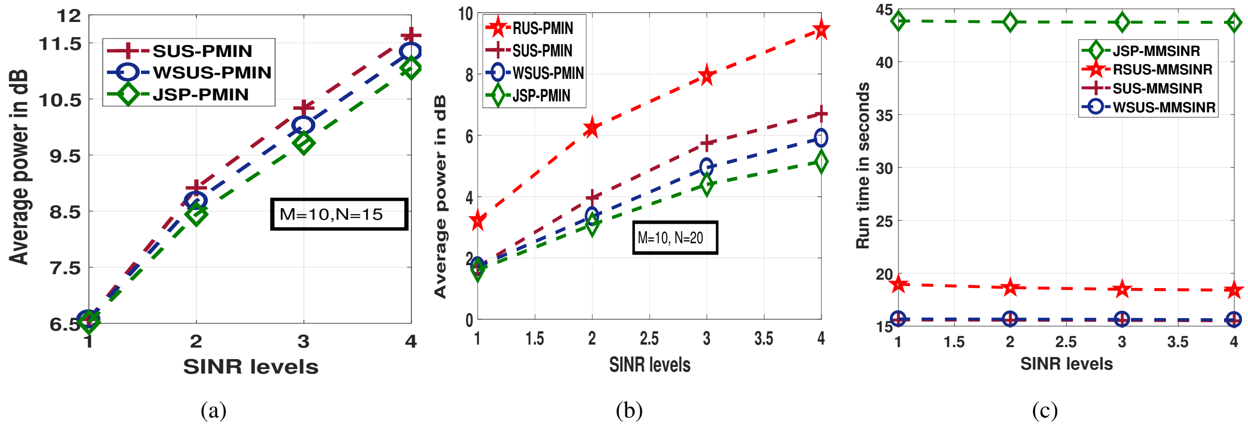


Fig. 5: Comparison of different PMIN optimization approaches for $M = 10$, $P_T = 10$ dB and SINR levels varying from 1 to 4 (a) $N = 15$ (b) $N = 20$ (c) algorithm run time.

$N = 15$ in figure 5a and $N = 20$ in figure 5b. The SINR level 1, 2, 3 and 4 (chosen differently than MMSINR design) on the x-axis indicate that $\tilde{\epsilon}_i$ is randomly chosen from the sets $\{1\}$, $\{1, 2\}$, $\{1, 2, 3\}$ and $\{1, 2, 3, 4\}$ for user i respectively. For example, for the SINR level 2, $\tilde{\epsilon}_i$ for user i is randomly chosen from the set $\{1, 2\}$.

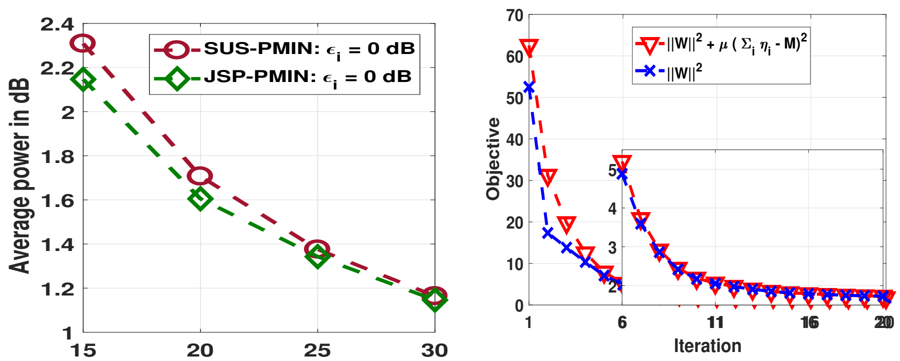
Figure 5a and 5b clear shows that the joint solution JSP-PMIN outperforms RUS-PMIN, SUS-PMIN and WSUS-PMIN. Although the precoding problem for the scheduled users by RUS, SUS, and WSUS is solved globally using [7], the inefficient scheduling leads to poorer performance compared to JSP-PMIN. On the contrary, the system design in JSP-PMIN helps to gain up to 25% (SINR level 4, figure 5b) in comparison with WSUS-PMIN. In figure 5c, the run time of algorithms is illustrated in seconds as a function of SINR levels. As shown in figure 5c, the performance gains of JSP-MMSINR incur higher computational complexity.

In table IV, the performance of JSP-PMIN and WSUS-PMIN is compared with ES-PMIN for $M = 5$, $\tilde{\epsilon}_i \in \{0, 1, 2, 3\}, \forall i$ for different N. Despite the theoretical guarantees of convergence JSP-PMIN only to a stationary point, JSP-PMIN performs close to ES-PMIN as can be observed in table IV. This justifies the efficiency of JSP-PMIN approach.

The performance JSP-PMIN for uniform weighted case (i.e., all users with same minimum SINR requirement) is illustrated in figure 6 for $M = 10$ and $\{\tilde{\epsilon}_i = 1\}_{i=1}^{N}$. In figure 6a, the average total power per CR in dB is depicted as a function of N varying from 15 to 30 in steps of 5. The superior performance of JSP-PMIN over SUS-PMIN is clear from figure 6a. However,

TABLE IV: Peformance comparison of JSP-PMIN with ES-PMIN and WSUS-PMIN for M =3, SINR level 4 and N is varied from 5 to 7 insteps of 1.

Number of users in a cell (N)	Average power consumed in dB with		
	ES-PMIN	JSP-PMIN	WSUS-PMIN
$N = 5$	5.4477	5.99	6.2448
$N = 6$	5.116	5.6164	5.8771
$N = 7$	3.95	4.5365	5.1270



(a) (b)

Fig. 6: Comparison of different PMIN optimization approaches for $M = 10$, $P_T = 10$dB and $\{\epsilon_i = 0\text{dB}\}_{i=1}^{N}$ (a) N varying from 10 to 30 in steps of 5 (b) convergence of the JSP-PMIN (with penalty) and convergence of $\boldsymbol{\eta}$ to binary for $N = 15$.

the gains diminish as N increases as the SUS based scheduling becomes efficient (kindly refer to similar discussion on WSR results).

In figure 6b, the convergence behavior of the JSP-PMIN algorithm (red curve) and the progression towards ensuring the exact scheduling constraint is depicted as a function of iterations for $N = 15$. The FIP may include the solutions that violate exact scheduling constraint due to which the penalized objective and objective differs by a large factor in the initial iterations. However, the increment in the penalty parameter μ ensures the exact scheduling constraint over the iterations. This is confirmed by figure 6b, as the difference between penalized objective and objective, becomes approximately zero. For the reasons at the beginning of this section, $\lambda_3 = 0$ still achieves the binary nature of $\boldsymbol{\eta}$ over iterations.

Heterogeneous users: Although homogeneous users with same channel characteristics for all the users are considered for simulations, the proposed framework is readily applicable to heterogeneous users as well. Moreover, heterogeneous users channels are relatively less orthogonal and channels' strength may vary greatly depending on user location. Hence, decoupled methods which employ the channel orthogonality based scheduling yields low-quality solutions. On the contrary, due to the systematic joint design of scheduling and precoding, the gains obtained by the proposed joint methods can be larger for heterogeneous users than homogeneous users.

VII. Conclusions

In this paper, the joint scheduling and precoding problem was considered for multiuser MISO downlink channels for three different network performance optimization criteria: weighted sum rate maximization, maximization of minimum SINR and power minimization. Unlike the existing works, the design is formulated in a way that is amenable to the joint update of scheduling and precoding. Observing that the original optimization to be an instance of the MINLP problem for the three considered criteria, the paper proposed efficient reformulations and relaxations to transform these into structured DC programming problems. Subsequently, the paper proposed joint scheduling and precoding CCP based algorithms (JSP-WSR, JSP-MMSINR, and JSP-PMIN) which are guaranteed to converge to a stationary point for the aforementioned DC problems. Finally, the paper proposed a low-complexity procedure to obtain a good feasible initial point, critical to the implementation of CCP based algorithms. Through simulations, the paper established the efficacy of the proposed joint techniques with respect to the decoupled benchmark solutions.

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